[a] $\frac{dr}{d\theta} = \frac{\cos r \sin\theta}{1+\sin r\cos\theta}$

FINAL ANSWER: $\cos\theta + \sin r = C\cos r$

TOTAL 12

ALL ITEMS 1 POINT ON ALL QUESTIONS UNLESS OTHERWISE INDICATED

$\boxed{-\sin\theta\cos r\,d\theta + (1+\cos\theta\sin r)dr = 0}$

$M = -\sin\theta\cos r \Rightarrow \boxed{M_r = \sin\theta\sin r}$ $\frac{1}{2}$

$N = 1+\cos\theta\sin r \Rightarrow \boxed{N_\theta = -\sin\theta\sin r}$ $\frac{1}{2}$

$\boxed{\frac{N_\theta - M_r}{M} = \frac{-2\sin\theta\sin r}{-\sin\theta\cos r} = \frac{2\sin r}{\cos r}}$ ⬅ FUNCTION OF ONLY r

$\boxed{\mu = e^{\int \frac{2\sin r}{\cos r}dr} = e^{-2\ln|\cos r|} = \sec^2 r}$

$\boxed{-\sin\theta\sec r\,d\theta + (\sec^2 r + \cos\theta\sec r\tan r)dr = 0}$

$M = -\sin\theta\sec r \Rightarrow \boxed{M_r = -\sin\theta\sec r\tan r}$

$N = \sec^2 r + \cos\theta\sec r\tan r \Rightarrow \boxed{N_\theta = -\sin\theta\sec r\tan r}$ ✓ ⬅ EXACT

$f = \int -\sin\theta\sec r\,d\theta \Rightarrow \boxed{f = \cos\theta\sec r + C(r)}$

$f_r = \boxed{\cos\theta\sec r\tan r + C'(r) = \sec^2 r + \cos\theta\sec r\tan r} \Rightarrow \boxed{C'(r) = \sec^2 r} \Rightarrow \boxed{C(r) = \tan r}$

$\frac{1}{2}$

$\frac{1}{2}$ $f = \boxed{\cos\theta\sec r + \tan r} = \boxed{C}$

$\boxed{\cos\theta + \sin r = C\cos r}$

[b] $x\,dy + (2xy - 2y - 4x^2\sqrt{y})dx = 0$ **FINAL ANSWER**: $y = (2x + Cxe^{-x})^2$

TOTAL ⑩

$$2xy - 2y - 4x^2\sqrt{y} + x\tfrac{dy}{dx} = 0$$

$\boxed{\tfrac{dy}{dx} + (2 - \tfrac{2}{x})y = 4x\sqrt{y}}$ ⬅ BERNOULLI $n = \frac{1}{2}$

Let $\boxed{v = y^{1-\frac{1}{2}} = y^{\frac{1}{2}}}$

So $\frac{dv}{dx} = \frac{1}{2}y^{-\frac{1}{2}}\frac{dy}{dx}$ and $\frac{dy}{dx} = 2y^{\frac{1}{2}}\frac{dv}{dx}$

$$\boxed{2y^{\frac{1}{2}}\tfrac{dv}{dx} + (2 - \tfrac{2}{x})y = 4x\sqrt{y}}$$

$$\tfrac{dv}{dx} + (1 - \tfrac{1}{x})y^{\frac{1}{2}} = 2x$$

$\boxed{\tfrac{dv}{dx} + (1 - \tfrac{1}{x})v = 2x}$ ⬅ LINEAR

$$\boxed{\mu = e^{\int(1-\frac{1}{x})dx} = e^{x-\ln|x|} = x^{-1}e^x}$$

$$\boxed{x^{-1}e^x\tfrac{dv}{dx} + (x^{-1} - x^{-2})e^x v = 2e^x}$$

$$\boxed{\text{CHECK: } \tfrac{d}{dx}x^{-1}e^x = -x^{-2}e^x + x^{-1}e^x \checkmark}$$

$$\boxed{x^{-1}e^x v = \int 2e^x dx = 2e^x + C}$$

$\boxed{v = 2x + Cxe^{-x}}$ (½)

$\boxed{y^{\frac{1}{2}} = 2x + Cxe^{-x}}$ (½)

$$\boxed{y = (2x + Cxe^{-x})^2}$$

[c] $t(t-x)x' - tx = x^2$ **FINAL ANSWER:** $xte^{\frac{t}{x}} = C$

TOTAL

(8½)

$$t(t-x)\tfrac{dx}{dt} - tx - x^2 = 0$$

$$(t^2 - xt)dx + (-tx - x^2)dt = 0$$

$$(kt)^2 - (kx)(kt) = k^2(t^2 - xt) \text{ and } -(kt)(kx) - (kx)^2 = k^2(-tx - x^2)$$ ⬅ HOMOGENEOUS

Let $x = vt$

$$(t^2 - vt^2)(vdt + tdv) + (-vt^2 - v^2t^2)dt = 0$$

$$(1-v)(vdt + tdv) + (-v - v^2)dt = 0$$

$$(-v - v^2 + v - v^2)dt + (1-v)tdv = 0$$

$$-2v^2dt + (1-v)tdv = 0$$

$$2v^2dt = (1-v)tdv$$

$$\int \tfrac{2}{t}dt = \int (v^{-2} - v^{-1})dv$$ ⬅ SEPARABLE

$$2\ln|t| + C = -v^{-1} - \ln|v|$$

$$2\ln|t| + C = -\tfrac{t}{x} - \ln\left|\tfrac{x}{t}\right|$$ (½)

$$2\ln|t| + C = -\tfrac{t}{x} - \ln|x| + \ln|t|$$

$$\ln|t| + C = -\tfrac{t}{x} - \ln|x|$$

$$Ct = e^{-\frac{t}{x}}x^{-1}$$

$$xte^{\frac{t}{x}} = C$$

ALTERNATE SOLUTION

GRADE USING ONLY ONE SOLUTION

[c] $t(t-x)x' - tx = x^2$ **FINAL ANSWER:** $xte^{\frac{t}{x}} = C$

$$t(t-x)\tfrac{dx}{dt} - tx - x^2 = 0$$

$$(t^2 - xt)dx + (-tx - x^2)dt = 0$$

$$(kt)^2 - (kx)(kt) = k^2(t^2 - xt) \text{ and } -(kt)(kx) - (kx)^2 = k^2(-tx - x^2)$$ ⬅ HOMOGENEOUS

Let $t = vx$

$$(v^2x^2 - vx^2)dx + (-vx^2 - x^2)(vdx + xdv) = 0$$

$$(v^2 - v)dx + (-v - 1)(vdx + xdv) = 0$$

$$(v^2 - v - v^2 - v)dx + (-v - 1)xdv = 0$$

$$-2vdx + (-v - 1)xdv = 0$$

$$-2vdx = (v + 1)xdv$$

$$\int -\tfrac{2}{x}dx = \int (1 + v^{-1})dv$$ ⬅ SEPARABLE

$$-2\ln|x| + C = v + \ln|v|$$

$$-2\ln|x| + C = \tfrac{t}{x} + \ln\left|\tfrac{t}{x}\right|$$ (½)

$$-2\ln|x| + C = \tfrac{t}{x} + \ln|t| - \ln|x|$$

$$C = \tfrac{t}{x} + \ln|t| + \ln|x|$$

$$xte^{\frac{t}{x}} = C$$